



Abstract

The quadrupole scan technique is a useful method used to calculate transverse beam emittance and characterize the beam phase space parameters in linear accelerators. This paper discusses the implementation of the technique at the Fermilab Accelerator Science and Technology (FAST) Facility. With the use of flexible Python scripts combined with Fermilab's control system ACNET, an operator will be permitted to select a quadrupole and analyzing screen of choice to quickly perform a quadrupole scan and then measure the beam emittance almost instantly. We also discuss the applicability of the quadrupole scan method at 20 MeV at an operating charge of 13 pC during FAST commissioning. Some preliminary measurements will also be presented.

FAST Electron Photoinjector

Parameter	ILC nominal	Range
Bunch charge	3.2 nC	10pC to > 20 nC
Bunch spacing	333 ns	<10 ns to 10 s
Bunch train	1 ms	1 bunch to 1 ms
Train rep. rate	5 Hz	0.1 Hz to 5 Hz
Transverse emit.	25 mm-mrad	1 to 100 mm-mrad
r.m.s. bunch length	1 ps	10fs to 10ps
Beam energy	300 MeV	50-300 MeV

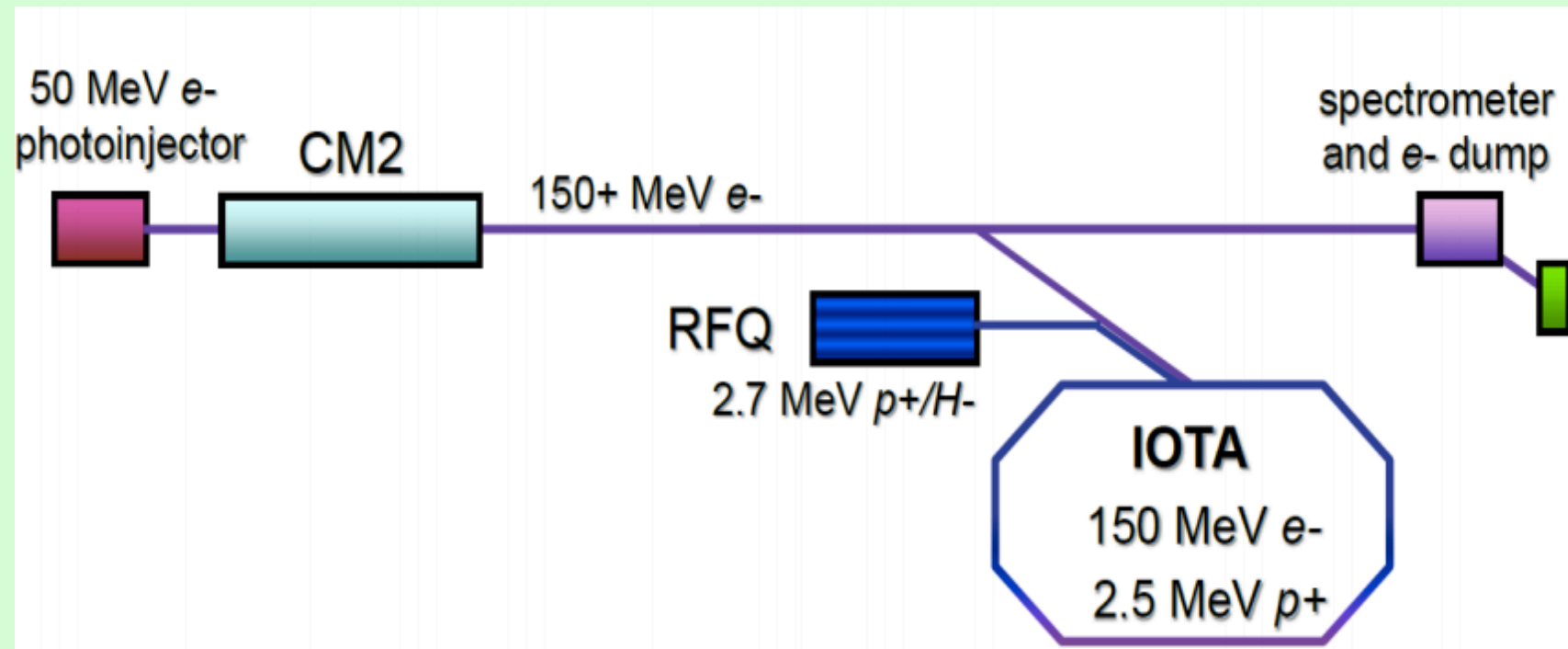


Fig.1: Beam with default quad settings (beam size < 3 mm).

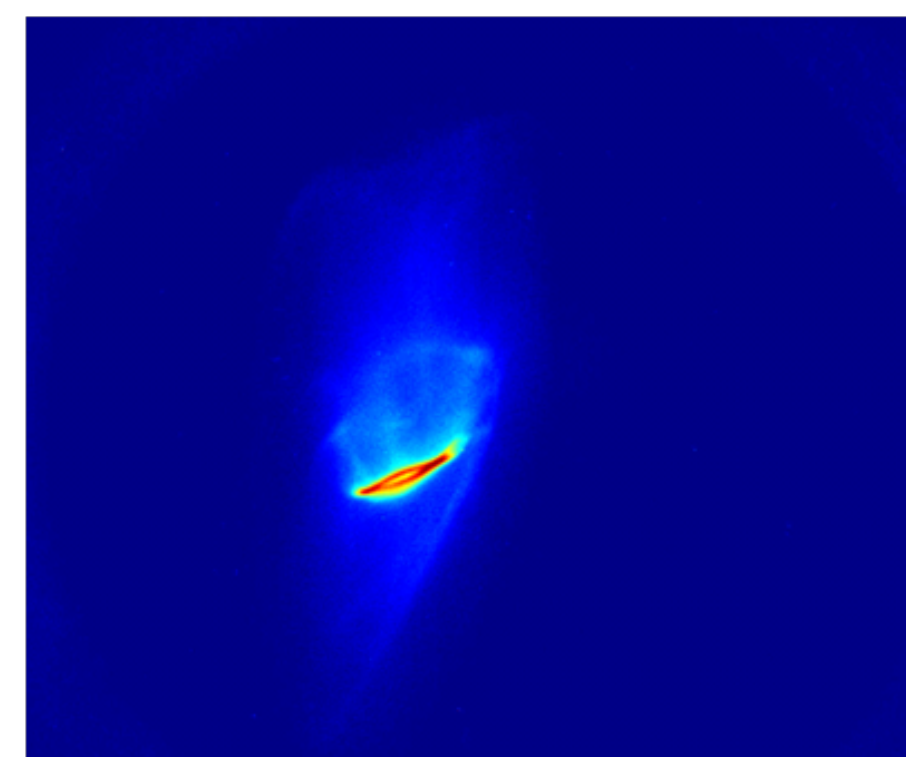


Fig.2: Focused beam (beam size < 100 μm).



Quadrupole Scan Technique at FAST

- Select a quadrupole magnet and imaging station (typically a YAG crystal or OTR screen).
- Vary the quad current, in turn changing the magnetic field strength, until the minimum spot size is found.
- Load quad scan Python script from Fermilab's ACNET console.
- Select magnetic field strength range and run Python script.
- Automated quad scan script yields averaged quad current settings (A) and beam sizes (pixels) via Gaussian fit in both horizontal and vertical planes in < 10 min.

Thin & Thick Lens Models

“Thin Lens” approximation:

- Plot beam size (squared) as a function of magnetic field strength.
- Apply 2nd order polynomial fit and retrieve coefficients (A, B, and C).
- Calculate sigma (beam) matrix elements from coefficients.
- Solve for the geometrical emittance and C-ε parameters.

$$R = S \cdot Q = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ k \cdot l & 1 \end{bmatrix}$$

$$\sigma_{11} = AK^2 + BK + C$$

$$\langle x^2 \rangle = \sigma_{11} = (S_{11} + KLS_{12})^2 \sigma_{11} + S_{12}^2 \sigma_{22} + 2S_{12}(S_{11} + KLS_{12})\sigma_{12}$$

$$\sigma_{11} = \frac{A}{l \cdot S_{12}^2}$$

$$\sigma_{12} = \sigma_{21} = \frac{B - 2\sigma_{11} \cdot l \cdot S_{11} \cdot S_{12}}{2l \cdot S_{12}^2}$$

$$\sigma_{22} = \frac{C - \sigma_{11} \cdot S_{11}^2 - 2\sigma_{12} \cdot S_{11} \cdot S_{12}}{S_{12}^2}$$

$$\epsilon_g = \sqrt{\sigma_{11q}\sigma_{22q} - \sigma_{12q}^2}$$

$$\beta = \frac{\sigma_{11}}{\epsilon_g}$$

$$\alpha = -\frac{\sigma_{12}}{\epsilon_g}$$

$$\gamma = \frac{1 + \alpha^2}{\beta}$$

“Thick Lens” model:

- Set the array of beam size measurements equal to the transfer matrix multiplied by the sigma matrix.
- Apply a least-squares fit to solve for the three sigma matrix elements.
- Again, solve for the geometrical emittance by taking the square root of the determinant of the beam matrix

$$R = S \cdot Q = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} \cos \phi & \frac{1}{\sqrt{|k|}} \sin \phi \\ -\sqrt{|k|} \sin \phi & \cos \phi \end{bmatrix}$$

$$R_{11} = S_{11} \cos \phi - S_{12} \sqrt{|k|} \sin \phi$$

$$R_{12} = S_{11} \frac{1}{\sqrt{|k|}} \sin \phi + S_{12} \cos \phi$$

$$\begin{bmatrix} \sigma_{11s}^{(a)} \\ \sigma_{11s}^{(b)} \\ \vdots \\ \sigma_{11s}^{(n)} \end{bmatrix} = \begin{bmatrix} R_{11}^{2(a)} & 2R_{11}^{2(a)}R_{12}^{2(a)} & R_{12}^{2(a)} \\ R_{11}^{2(b)} & 2R_{11}^{2(b)}R_{12}^{2(b)} & R_{12}^{2(b)} \\ \vdots & \vdots & \vdots \\ R_{11}^{2(n)} & 2R_{11}^{2(n)}R_{12}^{2(n)} & R_{12}^{2(n)} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{11q} \\ \sigma_{12q} \\ \sigma_{22q} \end{bmatrix}$$

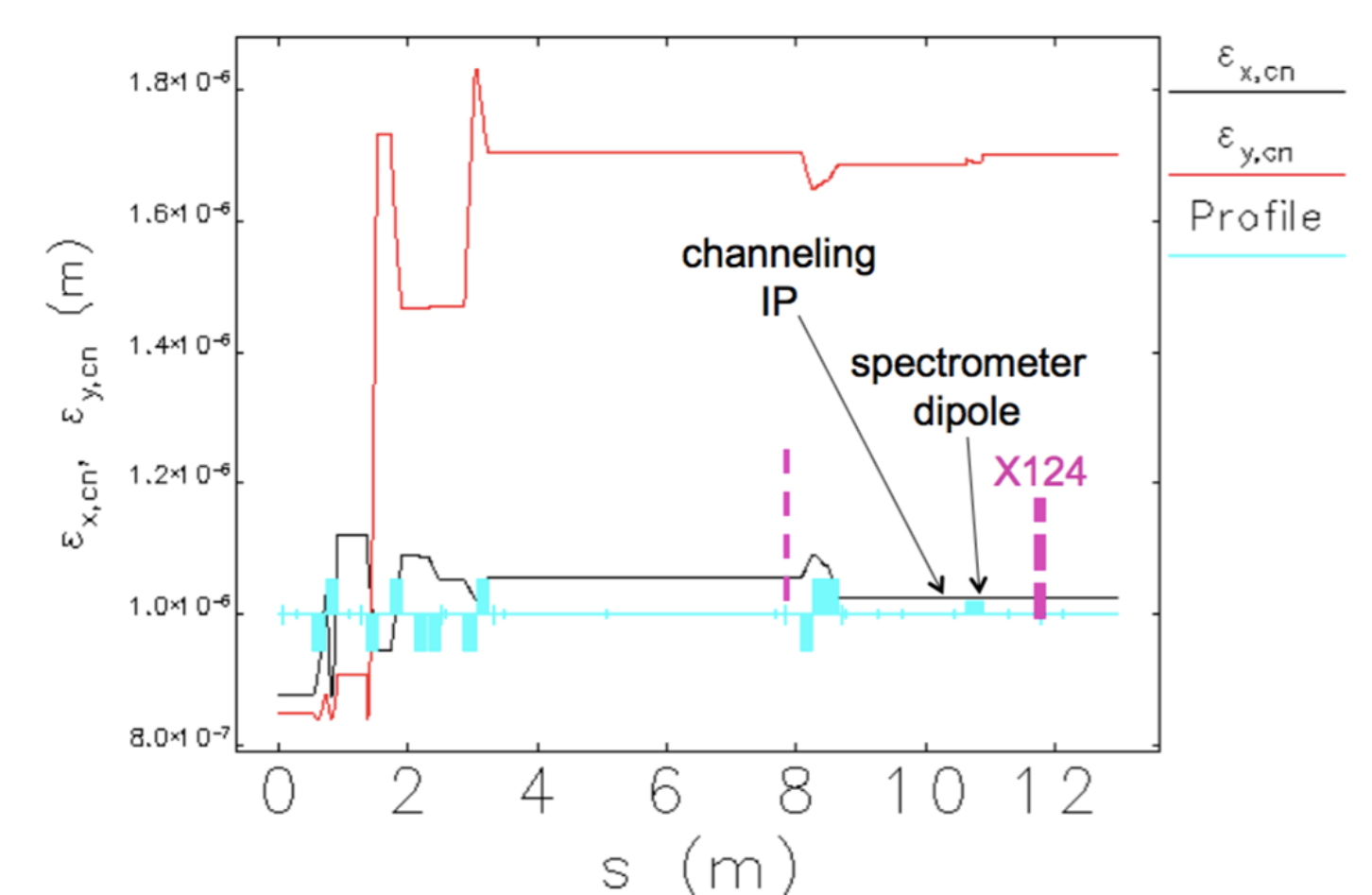
$$= A \cdot \begin{bmatrix} \sigma_{11q} \\ \sigma_{12q} \\ \sigma_{22q} \end{bmatrix}$$

I would like to thank D. Broemmelsiek, K. Carlson, D. Crawford, D. Edstrom, A. Lumpkin, P.R.G. Piot, A. Romanov, J. Ruan, J. Santucci, G. Stancari, C. Thangaraj, and A. Valishev of Fermi National Accelerator Lab (FNAL) for the technical support for quad-scan tests of the injector beamline at the Fermilab Accelerator Science and Technology facility (FAST).

Simulation/Predicted Results

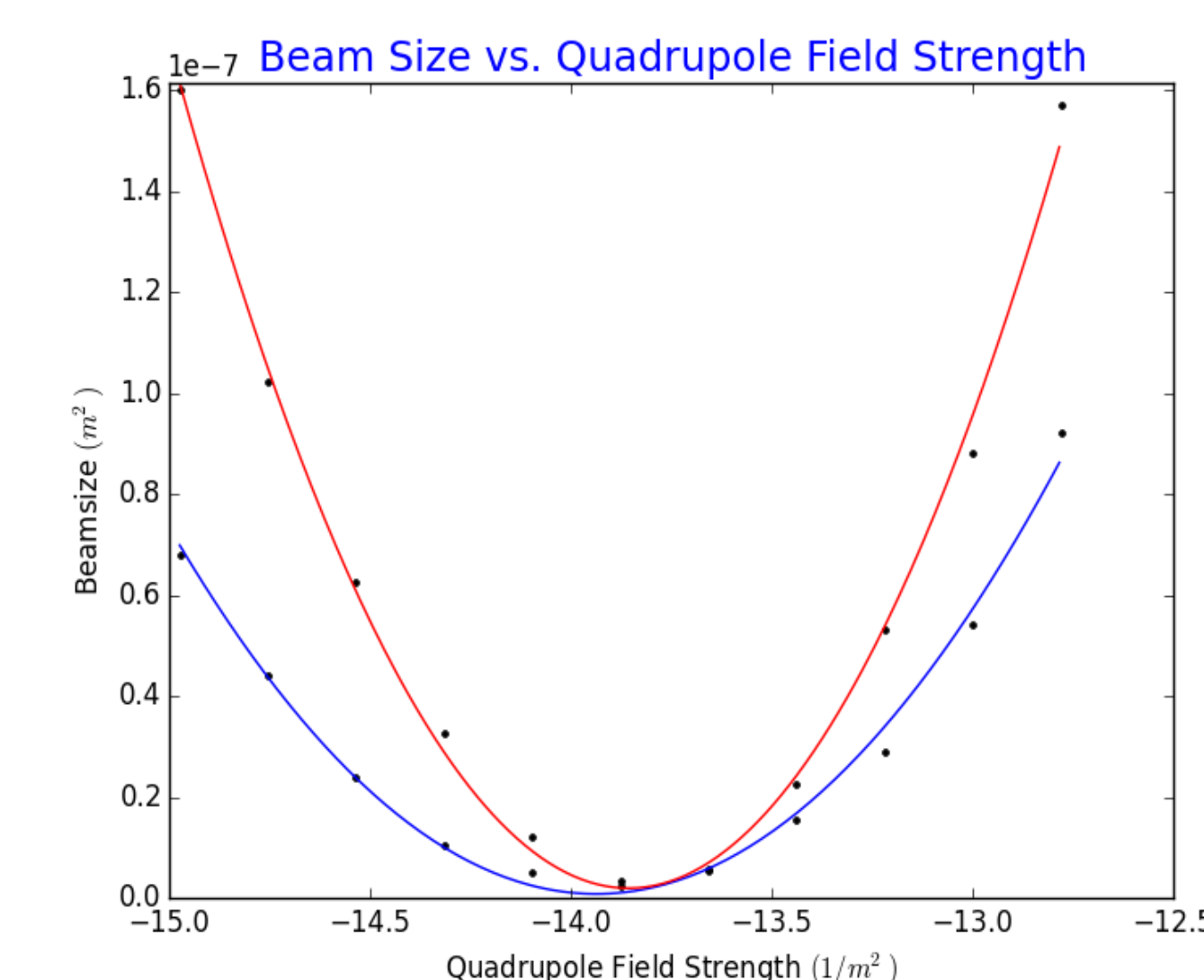
- Simulation performed using ELEGANT particle tracking code.
- Beam energy: 25 MeV
- Bunch charge: 250 pC

Emittance evolution downstream of CAV2



Preliminary Experimental Results

- While still in the commissioning stage, a quad scan was performed to test our preliminary version of the automated quad scan Python script and “Thin Lens” emittance calculator.



- Left is a plot of beam size (squared) vs. quad field strength.

- Below are the beam parameters and preliminary calculated results from experiment.

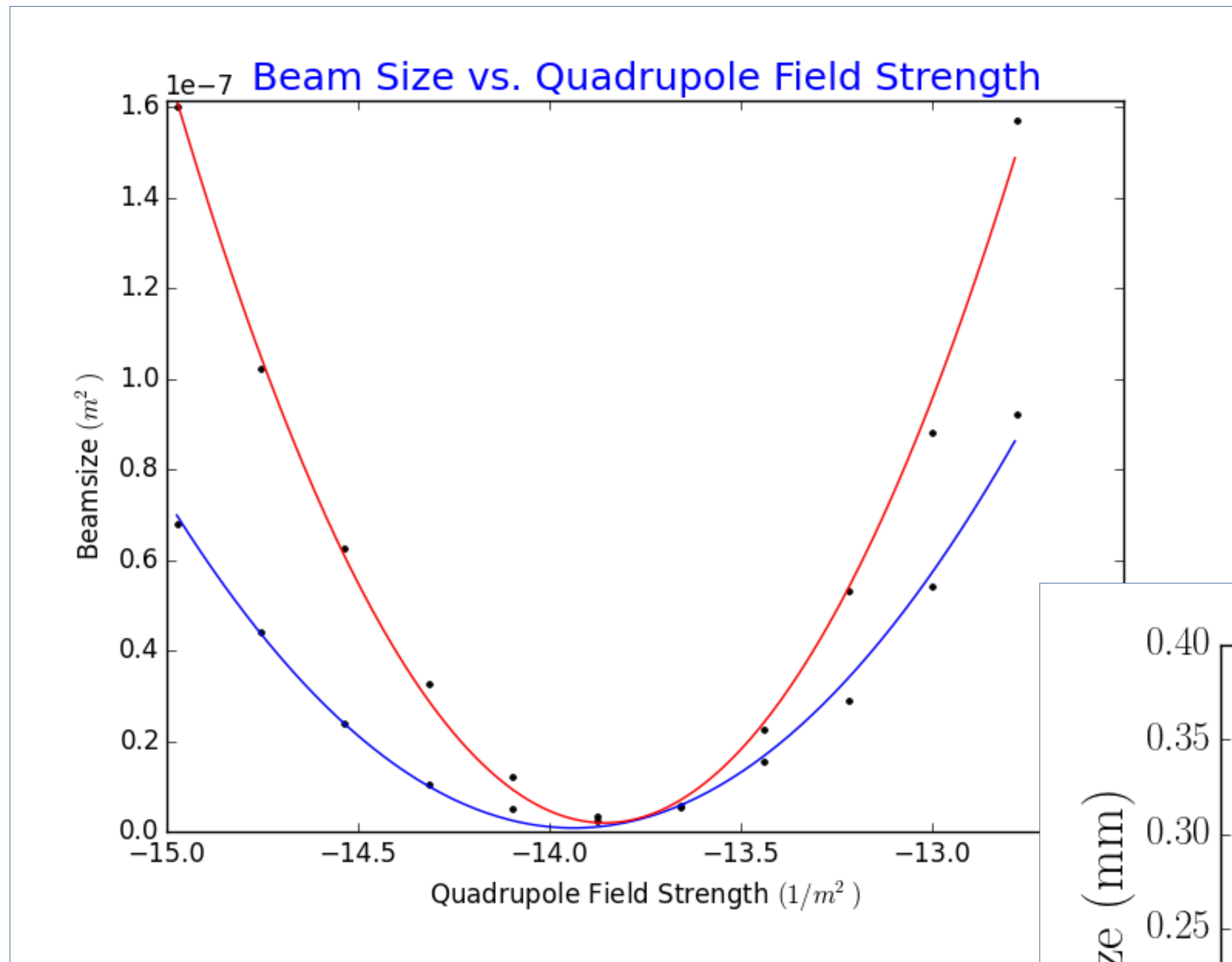
Horizontal Plane		Vertical Plane	
Parameter	Value	Parameter	Value
α	77.94	α	-70.04
β	53.85	β	48.85
ε (geometrical)	0.0332 mm*mrad	ε (geometrical)	0.0727 mm*mrad
ε (normalized)	1.35 mm*mrad	ε (normalized)	2.96 mm*mrad

Parameter	Value
Beam Energy	20.3 MeV
Bunch Charge	12-14 pC
Drift Length	1.14 m



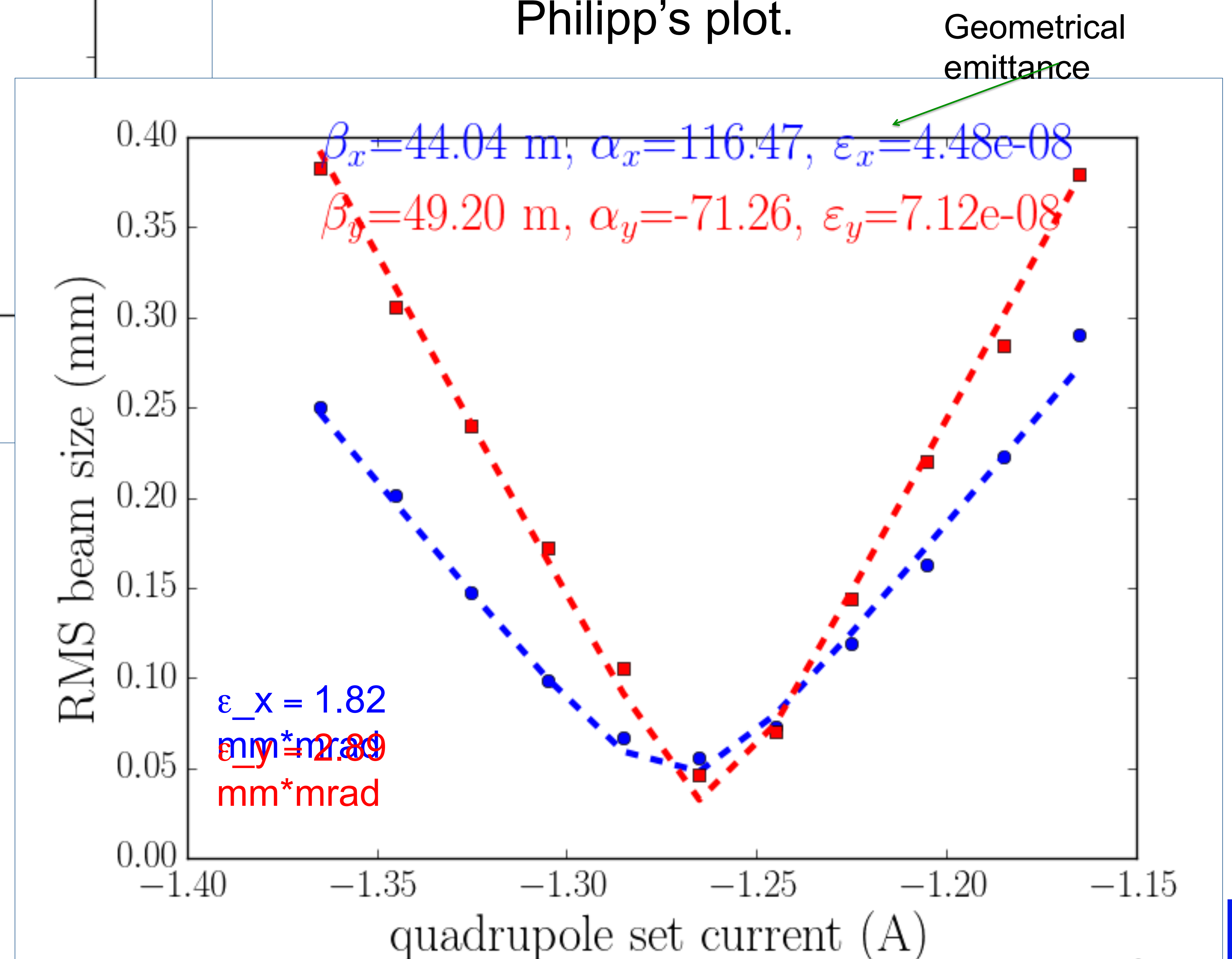
Q120/X121 Comparison

My plot.



Parameter	Value
Beam Energy	20.3 MeV
Bunch Charge	12-14 pC
Drift Length	1.14 m

Philipp's plot.



Horizontal Plane

Vertical Plane

Parameter	Value	Parameter	Value
α	77.94	α	-70.04
β	53.85	β	48.85
ϵ (geometrical)	0.0332 mm*mrad	ϵ (geometrical)	0.0727 mm*mrad
ϵ (normalized)	1.35 mm*mrad	ϵ (normalized)	2.96 mm*mrad



Emittance Comparison

Beam parameters: 20.3 MeV beam energy, ~13 pC, 1.136037 m drift length.

File name: X121_20150601_164403/sigma.dat

“Thin Lens” approximation using 2nd order polynomial fit with Python:

$\epsilon_{gx}, \epsilon_{gy} \text{ (mm} \cdot \text{mrad)}$	0.0332, 0.0727
$\epsilon_{nx}, \epsilon_{ny} \text{ (mm} \cdot \text{mrad)}$	1.35, 2.96
α_x, α_y	77.94, -70.04
$\beta_x, B_y \text{ (m)}$	53.85, 48.85

“Thick Lens” model using least-squares fit with Python:

$\epsilon_{gx}, \epsilon_{gy} \text{ (mm} \cdot \text{mrad)}$	0.0301, 0.0709
$\epsilon_{nx}, \epsilon_{ny} \text{ (mm} \cdot \text{mrad)}$	1.34, 2.89
α_x, α_y	72.83, -66.95
$\beta_x, B_y \text{ (m)}$	47.23, 43.85

Emittance/C-S parameters from Prof. Piot:

$\epsilon_{gx}, \epsilon_{gy} \text{ (mm} \cdot \text{mrad)}$	0.0448, 0.0712
$\epsilon_{nx}, \epsilon_{ny} \text{ (mm} \cdot \text{mrad)}$	1.82, 2.89
α_x, α_y	116.47, -71.26
$\beta_x, B_y \text{ (m)}$	44.04, 49.20